Q. No. 1 – 5 Carry One Mark Each

1. Which of the following options is the closest in meaning to the word underlined in the sentence

below? In a democracy, everybody has the freedom to <u>disagree</u> with the government.

(A) Dissent (B) Descent (C) Decent (D) Decadent

Answer: A

Exp: Dissent is to disagree

2. After the discussion, Tom said to me, 'Please revert!' He expects me to _________.

(A) Retract (B) Get back to him

(C) Move in reverse (D) Retreat

Answer: B

Exp: Revert means set back

3. While receiving the award, the scientist said, "I feel vindicated". Which of the following is closest in meaning to the word 'vindicated'?

(A) Punished (B) Substantiated (C) Appreciated (D) Chastened

Answer: B

Exp: Vindicate has 2 meanings

1. Clear of blain
2. Substantiate, justify

4. Let $f(x,y) = x^n y^m = P$. If x is doubled and y is halved, the new value of f is

(A) $2^{n-m}P$ (B) $2^{m-n}P$ (C) $2(n-m)P$ (D) $2(m-n)P$

Answer: A

Exp: $(2x)^n \times \left(\frac{y}{2}\right)^m = 2^{n-m} \times x^n y^m$

5. In a sequence of 12 consecutive odd numbers, the sum of the first 5 numbers is 425. What is the sum of the last 5 numbers in the sequence?

Answer: 495

Exp: Let consecutive odd numbers be a-10, a-8, a-6, a-4, a-2, a, ……a+12

Sum of 1^{st} 5 number $= 5a-30=425 \Rightarrow a=91$

Last 5 numbers=(a+4)+(a+6)+…….+(a+12)

=(95+97+99+101+103)= $\boxed{495}$

Q. No. 6 – 10 Carry Two Marks Each

6. Find the next term in the sequence: 13M, 17Q, 19S, ___

(A) 21W (B) 21V (C) 23W (D) 23V

Answer: C

Exp:

$13 \qquad M$

$17(13+4) \quad Q(M+4)$

$19(17+2) \quad S(Q+2)$

$23(19+4) \quad W=(s+4)$

$\Rightarrow 23W$

7. If 'KCLFTSB' stands for 'best of luck' and 'SHSWDG' stands for 'good wishes', which of the following indicates 'ace the exam'?

(A) MCHTX (B) MXHTC (C) XMHCT (D) XMHTC

Answer: B

Exp: KCLFTSB SHSWDG

Reverse order: Reverse order:

BCS TOF LUCK GO OD W I S HES

Ace the exam

Reverse order should be

MAXE EHT ECA

Looking at the options we have M X H T C

8 . Industrial consumption of power doubled from 2000-2001 to 2010-2011. Find the annual rate of increase in percent assuming it to be uniform over the years.

(A) 5.6 (B) 7.2 (C) 10.0 (D) 12.2

Answer: B

Exp:

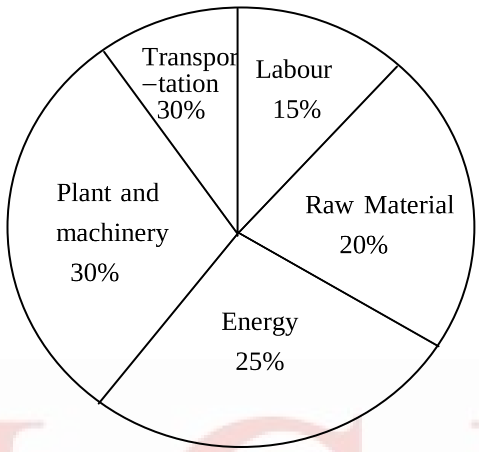
$$A=P\left(1+\frac{r}{100}\right)^n$$

$$A=2P$$

$$2=\left(1+\frac{r}{100}\right)^{10}$$

$$\therefore r=7.2$$

9. A firm producing air purifiers sold 200 units in 2012. The following pie chart presents the share of raw material, labour, energy, plant & machinery, and transportation costs in the total manufacturing cost of the firm in 2012. The expenditure on labour in 2012 is Rs. 4,50,000. In 2013, the raw material expenses increased by 30% and all other expenses increased by 20%. What is the percentage increase in total cost for the company in 2013?



Answer: 22%

Exp: Let total cost in 2012 is 100

Raw material increases in 2013 to 1.3 x 20=26

Other Expenses increased in 2013 to 1.2 x 80=96

Total Cost in 2013 =96+26=122

Total Cost increased by 22%

Hint:Labour cost (i.e, 4,50,000) in 2012 is redundant data.

10. A five digit number is formed using the digits 1,3,5,7 and 9 without repeating any of them. What is the sum of all such possible five digit numbers?

(A) 6666660 (B) 6666600 (C) 6666666 (D) 6666606

Answer: B

Exp: 1 appears in units place in 4! Ways

Similarly all other positions in 4! Ways

Same for other digits.

Sum of all the numbers = (11111) X 4! (1+3+5+7+9) = 6666600

Q.No. 1 – 25 Carry One Mark Each

1. The series $\sum_{n=0}^{\infty}\frac{1}{n!}$ converges to

(A) 2 ln2 (B) $\sqrt{2}$ (C) 2 (D) *e*

Answer: D

Exp: $\sum_{n=0}^{\infty}\frac{1}{n!}=1+\frac{1}{1!}+\frac{1}{2!}+.........$

$$=e \text{ as } e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+.......,\forall\ x \text{ in } R$$

2. The magnitude of the gradient for the function $f(x,y,z)=x^2+3y^2+z^3$ at the point $(1,1,1)$ is _________.

Answer: 7

Exp: $(\nabla f)_{P(1,1,1)}=\left(\vec{i}(2x)+\vec{j}(6y)+\vec{k}(3z^2)\right)_{P(1,1,1)}$

$$=2\vec{i}+6\vec{j}+3\vec{k}$$

$$\left|(\nabla f)_P\right|=\sqrt{4+36+9}=7$$

3. Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to _____

Answer: 0.8

Exp: $X\sim N(0,1)\Rightarrow f(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2},-\infty<x<\infty$

$$\therefore E\{|x|\}=\int_{-\infty}^{\infty}|x|.f(x)\,dx$$

$$=\frac{1}{\sqrt{2\pi}}x^2\int_0^{\infty}x\,e^{-x^2/2}dx$$

$$=\frac{2}{\sqrt{2\pi}}\int_0^{\infty}e^{-u}du=\sqrt{\frac{2}{\pi}}=0.797\simeq 0.8$$

4. If *a* and *b* are constants, the most general solution of the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0 \text{ is}$$

(A) ae^{-t} (B) $ae^{-t}+bte^{-t}$ (C) $ae^{t}+bte^{-t}$ (D) ae^{-2t}

Answer: B

Exp: $A.E: -m^2+2m+1=0\Rightarrow m=-1,-1$

$\therefore$ general solution is $x=(a+bt)e^{-t}$

5. The directional derivative of $f(x,y)=\frac{xy}{\sqrt{2}}(x+y)$ at $(1,1)$ in the direction of the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, is given by _____ .

Answer: 3

Exp: $f=\frac{1}{\sqrt{2}}\left(x^2y+xy^2\right)\Rightarrow \nabla f=\vec{i}\left[\frac{2xy+y^2}{\sqrt{2}}\right]+\vec{j}\left[\frac{x^2+2xy}{\sqrt{2}}\right]$

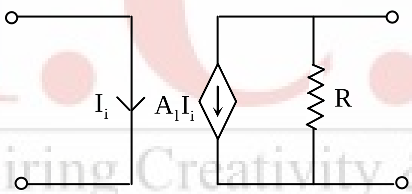
at $(1,1),\nabla f=\frac{3}{\sqrt{2}}\vec{i}+\frac{3}{\sqrt{2}}\vec{j}$

$\hat{e}=$ unit vector in the direction i.e., making an angle of $\frac{\pi}{4}$ with y-axis

$=\left(\sin\frac{\pi}{4}\right)\vec{i}+\left(\cos\frac{\pi}{4}\right)\vec{j}$

$\therefore$ directional derivative $=\hat{e}.\nabla f=2\left(\frac{3}{\sqrt{2}}\right)\left(1/\sqrt{2}\right)=3$

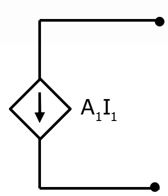
6. The circuit shown in the figure represents a



(A) Voltage controlled voltage source
(B) Voltage controlled current source
(C) Current controlled current source
(D) Current controlled voltage source

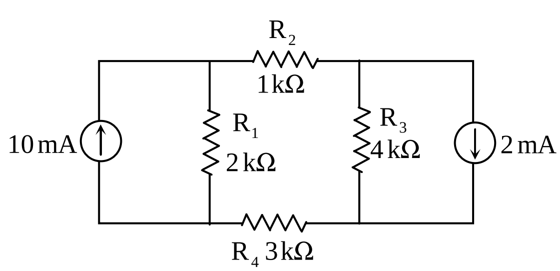
Answer: C

Exp:



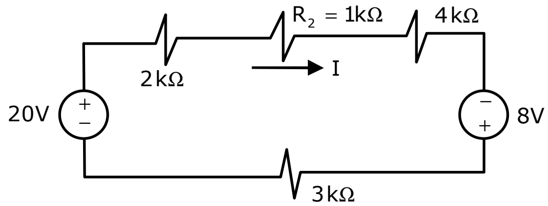
The dependent source represents a current controlled current source

7. The magnitude of current (in mA) through the resistor R_2 in the figure shown is_______.



Answer: 2.8

Exp: By source transformation



By KVL,

$20 - 10k.I + 8 = 0$

$\Rightarrow I = \dfrac{28}{10k}$

$\Rightarrow \boxed{I = 2.8mA}$

8. At T = 300 K, the band gap and the intrinsic carrier concentration of GaAs are 1.42 eV and 10^6 cm^{-3}, respectively. In order to generate electron hole pairs in GaAs, which one of the wavelength (λ_C) ranges of incident radiation, is most suitable? (Given that: Plank's constant is 6.62 × 10^{-34} J-s, velocity of light is 3 × 10^{10} cm/s and charge of electron is 1.6 × 10^{-19} C)

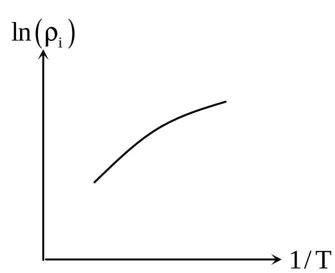
(A) 0.42 μm < λ_C < 0.87 μm (B) 0.87 μm < λ_C < 1.42 μm

(C) 1.42 μm < λ_C < 1.62 μm (D) 1.62 μm < λ_C < 6.62 μm

Answer: A

Exp: $E = \dfrac{hC}{\lambda} \Rightarrow \lambda = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{1.42\times1.6\times10^{-19}} = 0.87\mu m$

9. In the figure $\ln(\rho_i)$ is plotted as a function of 1/T, where ρ_i the intrinsic resistivity of silicon, T is is the temperature, and the plot is almost linear.



The slope of the line can be used to estimate

(A) Band gap energy of silicon (Eg)

(B) Sum of electron and hole mobility in silicon $(\mu_n + \mu_p)$

(C) Reciprocal of the sum of electron and hole mobility in silicon $(\mu_n + \mu_p)^{-1}$

(D) Intrinsic carrier concentration of silicon (n_i)

Answer: A

Exp: $n_i \alpha\, T^{3/2} e^{-Eg/kT}$ and

$\rho_i \;\alpha\; \dfrac{1}{\eta_i}$

$\therefore$ From the graph, Energy graph of S_i can be estimated

10. The cut-off wavelength (in μm) of light that can be used for intrinsic excitation of a semiconductor material of bandgap Eg= 1.1 eV is ________

Answer: 1.125

Exp: $E = \dfrac{hC}{\lambda}$

$$\Rightarrow \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.1\times1.6\times10^{-19}} = 1.125\mu m$$

11. If the emitter resistance in a common-emitter voltage amplifier is not bypassed, it will

(A) Reduce both the voltage gain and the input impedance

(B) Reduce the voltage gain and increase the input impedance

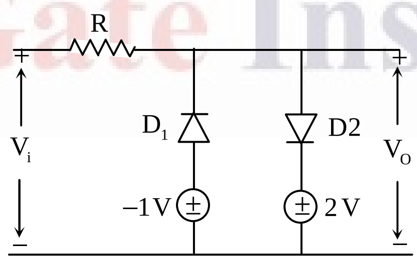
(C) Increase the voltage gain and reduce the input impedance

(D) Increase both the voltage gain and the input impedance

Answer: B

Exp: When a CE amplifier's emitter resistance is not by passed, due to the negative feedback the voltage gain decreases and input impedance increases

12. Two silicon diodes, with a forward voltage drop of 0.7 V, are used in the circuit shown in the figure. The range of input voltage V_i for which the output voltage $V_0 = V_i$, is



(A) $-0.3V < V_i < 1.3V$

(B) $-0.3V < V_i < 2V$

(C) $-1.0V < V_i < 2.0V$

(D) $-1.7V < V_i < 2.7V$

Answer: D

Exp: When $V_i < -1.7V$; $D_1 - ON$ and $D_2 - OFF$

$\therefore V_0 = -1.7V$

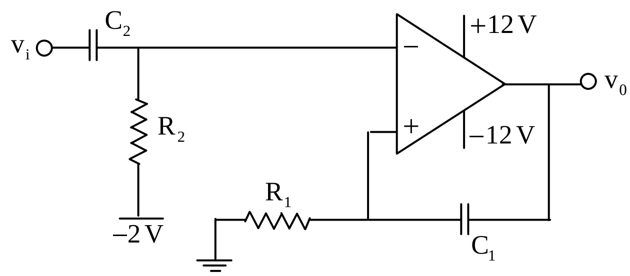
When $V_i > 2.7V$; $D_1 - OFF$ & $D_2 - ON$

$\therefore V_0 = 2.7V$

When $-1.7 < V_i < 2.7V$, Both D_1 & D_2 OFF

$\therefore V_0 = V_i$

13. The circuit shown represents:



(A) A bandpass filter
(B) A voltage controlled oscillator
(C) An amplitude modulator
(D) A monostable multivibrator

Answer: D

14. For a given sample-and-hold circuit, if the value of the hold capacitor is increased, then
(A) Droop rate decreases and acquisition time decreases
(B) Droop rate decreases and acquisition time increases
(C) Droop rate increases and acquisition time decreases
(D) Droop rate increases and acquisition time increases

Answer: B

Exp: Capacitor drop rate $= \frac{dv}{dt}$

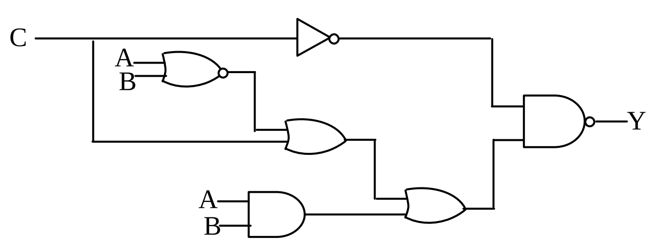
For a capacitor, $\frac{dv}{dt} \propto \frac{1}{c}$

$\therefore$ Drop rate decreases as capacitor value is increased

For a capacitor, $Q = cv = i \times t \Rightarrow t \propto c$

$\therefore$ Acquisition time increases as capacitor value increased

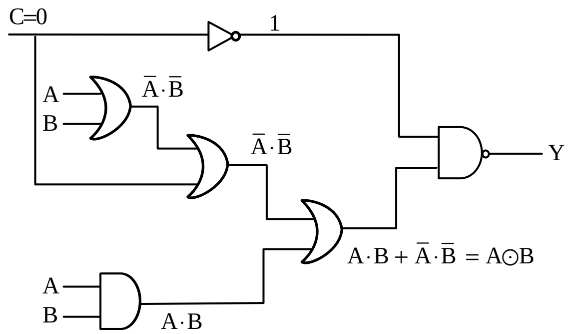
15. In the circuit shown in the figure, if C=0, the expression for *Y* is



(A) $Y = A\overline{B} + \overline{A}B$
(B) $Y = A + B$
(C) $Y = \overline{A} + \overline{B}$
(D) $Y = AB$

Answer: A

Exp:



$Y = \overline{1.A \odot B}$

$= \overline{A \odot B}$

$= A \oplus B = \bar{A}B + A\bar{B} + \bar{A}B$

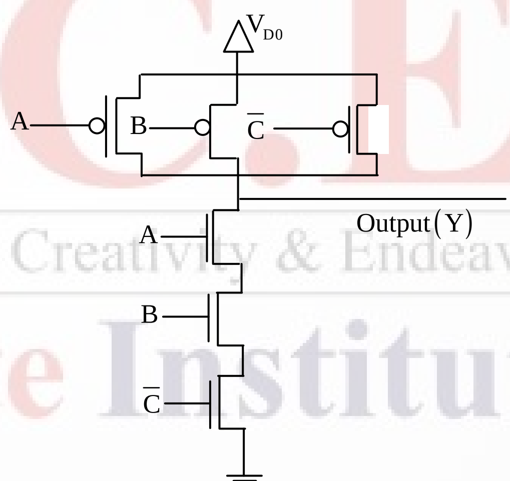
16. The output (Y) of the circuit shown in the figure is

(A) $\bar{A}+\bar{B}+C$

(B) $A+\bar{B}.\bar{C}+A.\bar{C}$

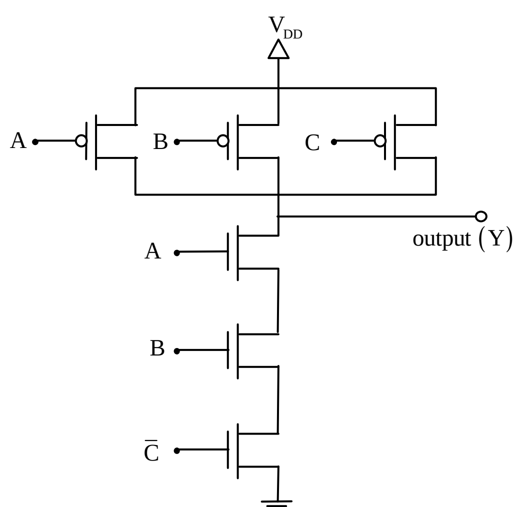
(C) $\bar{A}+B+\bar{C}$

(D) $A.B.\bar{C}$



Answer: A

Exp:



This circuit is CMOS implementation

If the NMOS is connected in series, then the output expression is product of each input with complement to the final product.

So, $Y = \overline{A.B.\overline{C}}$

$= \overline{A} + \overline{B} + C$

17. A Fourier transform pair is given by

$$\left(\frac{2}{3}\right)^n u[n+3] \overset{FT}{\Leftrightarrow} \frac{A e^{-j6\pi f}}{1-\left(\frac{2}{3}\right)e^{-j2\pi f}}$$

where $u[n]$ denotes the unit step sequence. The value of A is _________.

Answer: 3.375

Exp: $x[n] = \left(\frac{2}{3}\right)^n u[n+3]$

$$X\left(e^{j\Omega}\right) = \sum_{n=-3}^{\infty} \left(\frac{2}{3}\right)^n .e^{-j\Omega n} = \frac{\left(\frac{2}{3}\right)^{-3} .e^{j3\Omega}}{1-\frac{2}{3}e^{-j\Omega}}$$

$$\Rightarrow A = \left(\frac{3}{2}\right)^3 = \frac{27}{8} = 3.375$$

18. A real-valued signal $x(t)$ limited to the frequency band $|f| \le \frac{W}{2}$ is passed through a linear time invariant system whose frequency response is

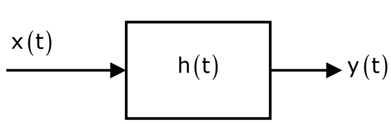
$$H(f) = \begin{cases} e^{-j4\pi f}, & |f| \le \frac{W}{2} \\ 0, & |f| > \frac{W}{2} \end{cases}$$

The output of the system is

(A) $x(t+4)$ (B) $x(t-4)$ (C) $x(t+2)$ (D) $x(t-2)$

Answer: D

Exp: Let $x(t)$ Fourier transform be $x(t)$



$y(t) = x(t)*h(t)[\text{convolution}]$

$\Rightarrow Y(f) = X(f).H(f)$

$\Rightarrow Y(f) = e^{-j4\pi f}.X(f)$

$\Rightarrow y(t) = x(t-2)$

19. The sequence $x[n] = 0.5^n\, u[n]$, where u[n] is the unit step sequence, is convolved with itself to obtain y[n]. Then $\sum_{n=-\infty}^{\infty} y(n)$ ________________.

Answer: 4

Exp: $y[n] = x[n] * x[n]$

Let $Y(e^{j\Omega})$ is F.T. pair with $y[n]$

$$\Rightarrow Y(e^{j\Omega}) = X(e^{j\Omega}).X(e^{j\Omega})$$

$$Y(e^{j\Omega}) = \frac{1}{1-0.5e^{-j\Omega}}.\frac{1}{1-0.5e^{-j\Omega}}$$

$$\text{also } Y(e^{j\Omega}) = \sum_{h=-\infty}^{\infty} y[n].e^{-j\Omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} y[n] = Y(e^{j0}) = \frac{1}{0.5}.\frac{1}{0.5} = 4$$

20. In a Bode magnitude plot, which one of the following slopes would be exhibited at high frequencies by a 4th order all-pole system?

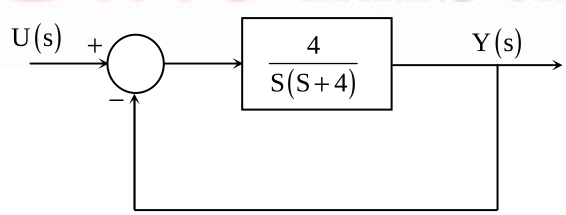
(A) – 80 dB/decade (B) – 40 dB/decade
(C) +40 dB/decade (D) +80 dB/decade

Answer: A

Exp: $\rightarrow$ In a BODE diagram, in plotting the magnitude with respect to frequency, a pole introduce a line 4 slope $-20\,\text{dB/dc}$

$\rightarrow$ If 4th order all-pole system means gives a slope of $(-20)*4\ \text{dB/dec}$ i.e. -80dB/dec

21. For the second order closed-loop system shown in the figure, the natural frequency (in rad/s) is



(A) 16 (B) 4 (C) 2 (D) 1

Answer: C

Exp: Transfer function $\frac{Y(s)}{U(s)} = \frac{4}{S^2+4s+4}$

If we compare with standard 2nd order system transfer function

i.e., $\frac{w_n^2}{s^2+2\xi w_n s + w_n^2}$

$w_n^2 = 4 \Rightarrow w_n = 2\ \text{rad/sec}$

22. If calls arrive at a telephone exchange such that the time of arrival of any call is independent of the time of arrival of earlier or future calls, the probability distribution function of the total number of calls in a fixed time interval will be

(A) Poisson (B) Gaussian (C) Exponential (D) Gamma

Answer: A

Exp: Poisson distribution: It is the property of Poisson distribution.

23. In a double side-band (DSB) full carrier AM transmission system, if the modulation index is doubled, then the ratio of total sideband power to the carrier power increases by a factor of __________________.

Answer: 4

Exp: $$\frac{\text{Ratio of total side band power}}{\text{Carrier power}} \alpha\, \mu^2$$

If it in doubled, this ratio will be come 4 times

24. For an antenna radiating in free space, the electric field at a distance of 1 km is found to be 12 mV/m. Given that intrinsic impedance of the free space is $120\pi\ \Omega$, the magnitude of average power density due to this antenna at a distance of 2 km from the antenna (in nW/m^2) is__________________.

Answer: 47.7

Exp: Electric field of an antenna is

$$E_\theta = \frac{\eta I_0 dl}{4\pi}\sin\theta\left[\underbrace{\frac{J\beta}{r}}_{\text{Radiation field}} + \underbrace{\frac{1}{r^2}}_{\text{inductive field}} - \underbrace{\frac{J}{\beta r^3}}_{\text{Electrostatic field}}\right]$$

$$\therefore E\ \alpha\ \frac{1}{r}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1} \Rightarrow E_2 = 6\,\text{mv/m}$$

$$P = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2}\frac{36\times10^{-8}}{120\pi} = 47.7\,\text{nw/m}^2$$

25. Match column A with column B.

Column A	Column B
(1) Point electromagnetic source	(P) Highly directional
(2) Dish antenna	(Q) End free
(3) Yagi-Uda antenna	(R) Isotropic

(A) $1\to P$, $2\to Q$, $3\to R$
(B) $1\to R$, $2\to P$, $3\to Q$
(C) $1\to Q$, $2\to P$, $3\to R$
(D) $1\to R$, $2\to Q$, $3\to P$

Answer: B

Exp: 1. Point electromagnetic source, can radiate fields in all directions equally, so isotropic.

2. Dish antenna $\rightarrow$ highly directional

3. Yagi – uda antenna $\rightarrow$ End fire

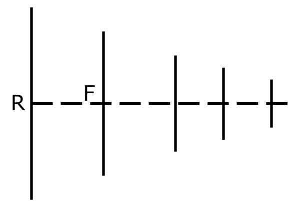


Figure: Yagi-uda antenna

Q. No. 26 – 55 Carry Two Marks Each

26. With initial values y(0) = y'(0)=1 the solution of the differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ at x = 1 is ________

Answer: 0.54

Exp: $A.E: m^2+4m+4=0 \Rightarrow m=-2,-2$

$\therefore$ solutions is $y=(a+bx)e^{-2x}$ ....(1)

$y'=(a+bx)(-2e^{-2x})+e^{-2x}(b)$ ....(2)

using $y(0)=1;\ y'(0)=1,\ (1)$ and (2) gives

$a=1$ and $b=3$

$\therefore y=(1+3x)e^{-2x}$

at $x=1,\ y=4e^{-2}=0.541\simeq 0.54$

27. Parcels from sender S to receiver R pass sequentially through two post-offices. Each post-office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is__________.

Answer: 0.44

Exp: Parcel will be lost if

a. it is lost by the first post office

b. it is passed by first post office but lost by the second post office

Prob(parcel is lost) = $\frac{1}{5}+\frac{4}{5}x\frac{1}{5}=\frac{9}{25}$

P (Parcel lost by second post if it passes first post office)= P (Parcel passed by first post office) x P(Parcel lost by second post office)

$= \frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$

Prob(parcel lost by 2nd post office | parcel lost)$= \frac{4/25}{9/25} = \frac{4}{9} = 0.44$

28. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. Which one of the following is the unilateral Laplace transform of $g(t) = t.\, f(t)$?

(A) $\frac{-s}{(s^2+s+1)^2}$ (B) $\frac{-(2s+1)}{(s^2+s+1)^2}$ (C) $\frac{S}{(s^2+s+1)^2}$ (D) $\frac{2S+1}{(s^2+s+1)^2}$

Answer: D

Exp: (1)

If $f(t) \leftrightarrow F(s)$

Then $tf(t) \leftrightarrow \frac{-d}{ds} F(s)$

$= \frac{-d}{ds}\left(\frac{1}{s^2+s+1}\right)$

$= -\frac{-(2s+1)}{(s^2+s+1)^2} = \frac{2s+1}{(s^2+s+1)^2}$

Exp: (2)

$F(s) = \frac{1}{s^2+s+1}$

$L[g(t) = t.f(t)] = -\frac{d}{ds}[F(s)]$ (using multiplication by t)

$= \frac{2s+1}{(s^2+s+1)^2}$

29. For a right angled triangle, if the sum of the lengths of the hypotenuse and a side is kept constant, in order to have maximum area of the triangle, the angle between the hypotenuse and the side is

(A) 12^O (B) 36^O (C) 60^O (D) 45^O

Answer: (C) (As per IIT Website)

Exp: Let x (opposite side), y (adjacent side) and z (hypotenuse) of a right angled triangle.

Given $Z + y = K$ (constant) ......(1) and angle between them say 'θ' then Area,

$A = \frac{1}{2}xy = \frac{1}{2}(z\sin\theta)(z\cos\theta) = \frac{z^2}{4}\sin 2\theta$

Now $(1) \Rightarrow z + z\sin\theta = k \Rightarrow z = \frac{k}{1+\sin\theta}$

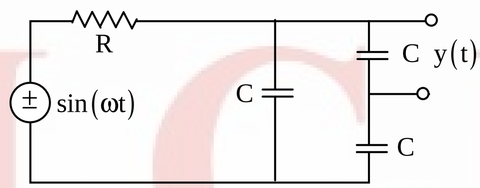
$$\therefore A = \frac{k^2}{4}\left[\frac{\sin 2\theta}{(1+\sin\theta)^2}\right]$$

In order to have maximum area, $\frac{dA}{d\theta} = 0$

$$\Rightarrow \frac{k^2}{4}\left[\frac{(1+\sin\theta)^2(2\cos 2\theta) - \sin 2\theta(\cos\theta).2(1+\sin\theta)}{(1+\sin\theta)^4}\right] = 0$$

$\Rightarrow \theta = \frac{\pi}{6} = 30^{o}$, Answer obtained is different than official key

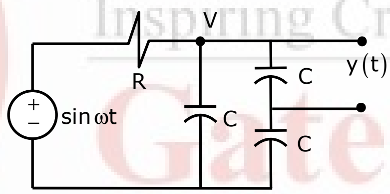
30. The steady state output of the circuit shown in the figure is given by $y(t) = A(\omega)\sin(\omega t + \phi(\omega))$. If the amplitude $|A(\omega)| = 0.25$, then the frequency ω is



(A) $\frac{1}{\sqrt{3}\,RC}$ (B) $\frac{2}{\sqrt{3}\,RC}$ (C) $\frac{1}{RC}$ (D) $\frac{2}{RC}$

Answer: B

Exp:



By nodal method, $\frac{V - 1\angle 0^{o}}{R} + \frac{V}{\left(1/j\omega c\right)} + \frac{V}{\left(2/j\omega c\right)} = 0$

$$V\left[\frac{1}{R} + j\omega c + \frac{j\omega c}{2}\right] = \frac{1\angle 0^{o}}{R}$$

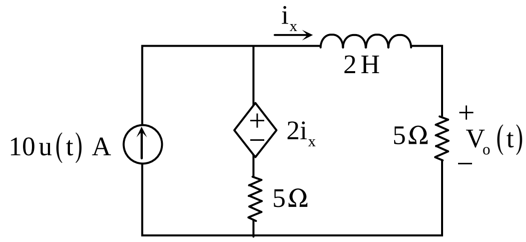
$$V = \frac{2}{2 + 3j\omega RC}$$

$$Y = \frac{V}{2} \Rightarrow \frac{1}{2 + j\omega 3RC}$$

$$\text{given } |A(\omega)| = \frac{1}{4} \Rightarrow \frac{1}{\sqrt{4 + 9R^2c^2.\omega^2}}$$

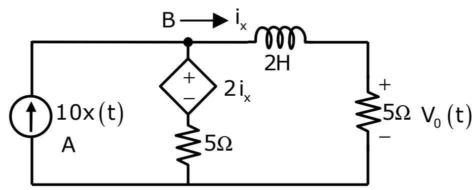
$$\Rightarrow \boxed{\omega = \frac{2}{\sqrt{3}\,RC}}$$

31. In the circuit shown in the figure, the value of $V_0(t)$ (in Volts) for $t \to \infty$ is ______.



Answer: 31.25

Exp:

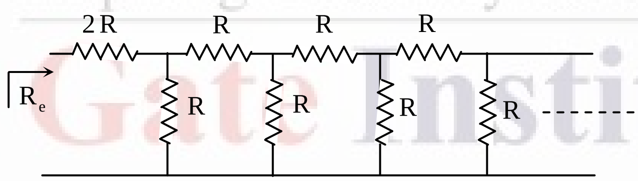


For $t \to \infty$, i.e., at steady state, inductor will behave as a shot circuit and hence $V_B = 5.i_X$

By KCL at node B, $-10 + V_B - 2i_x + i_x = 0 \Rightarrow i_x = \frac{50}{8}$

$V_0(t) = 5i_x(t) \Rightarrow V_0(t) = \frac{250}{8} = \boxed{31.25 \text{ volts}}$

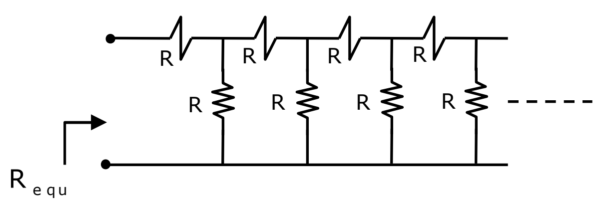
32. The equivalent resistance in the infinite ladder network shown in the figure is R_e.



The value of R_e/R is ________

Answer: 2.618

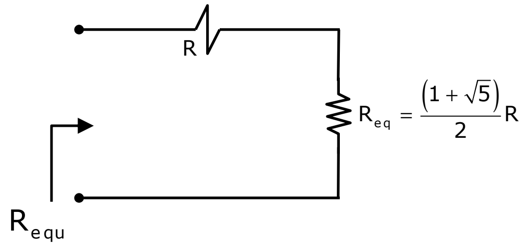
Exp:



$\rightarrow$ For an infinite ladder network, if all resistance are having same value of R

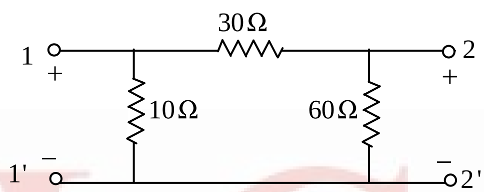
Then equivalent resistance is $\left(\frac{1+\sqrt{5}}{2}\right).R$

$\rightarrow$ For the given network, we can split in to R is in series with $R_{equivalent}$



$$\Rightarrow R_{equ} = R + 1.618R \Rightarrow \boxed{\frac{R_{equ}}{R} = 2.618}$$

33. For the two-port network shown in the figure, the impedance (Z) matrix (in Ω) is



(A) $\begin{bmatrix} 6 & 24 \\ 42 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} 9 & 8 \\ 8 & 24 \end{bmatrix}$ (C) $\begin{bmatrix} 9 & 6 \\ 6 & 24 \end{bmatrix}$ (D) $\begin{bmatrix} 42 & 6 \\ 6 & 60 \end{bmatrix}$

Answer: C

Exp: For the two-part network

$$\text{Y matrix} = \begin{bmatrix} \frac{1}{30}+\frac{1}{10} & -\frac{1}{30} \\ \frac{-1}{30} & \frac{1}{60}+\frac{1}{30} \end{bmatrix}$$

$$Z_{matrix} = [Y]^{-1}$$

$$Z = \begin{bmatrix} 0.1333 & -0.0333 \\ -0.0333 & 0.05 \end{bmatrix}^{-1}$$

$$Z = \begin{bmatrix} 9 & 6 \\ 6 & 24 \end{bmatrix}$$

34. Consider a silicon sample doped with $N_D = 1\times10^{15}/cm^3$ donor atoms. Assume that the intrinsic carrier concentration $n_i = 1.5\times10^{10}/cm^3$. If the sample is additionally doped with NA $= 1\times10^{18}/cm^3$ acceptor atoms, the approximate number of electrons/cm^3 in the sample, at T=300 K, will be ________________.

Answer: 225.2

Exp: $P = N_A - N_D = 1\times10^{18} - 1\times10^{15} = 9.99\times10^{17}$

$$\eta = \frac{\eta_i^2}{P} = \frac{(1.5\times10^{10})^2}{9.99\times10^{17}} = 225.2/cm^3$$

35. Consider two BJTs biased at the same collector current with area $A_1 = 0.2\mu m \times 0.2\mu m$ and $A_2 = 300\mu m \times 300\mu m$. Assuming that all other device parameters are identical, kT/q = 26 mV, the intrinsic carrier concentration is $1 \times 10^{10}\ cm^{-3}$, and $q = 1.6 \times 10^{-19}$ C, the difference between the base-emitter voltages (in mV) of the two BJTs (i.e., $V_{BE1} - V_{BE2}$) is ________________.

Answer: 381

Exp: $I_{C_1} = I_{C_2}$ (Given)

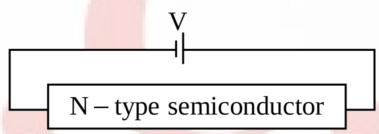
$$I_{S_1} e^{\frac{V_{BE_1}}{V_T}} = I_{S_2} e^{V_{BE_2}/V_T}$$

$$e^{(V_{BE_1} - V_{BE_2})/V_T} = \frac{I_{S_2}}{I_{S_1}}$$

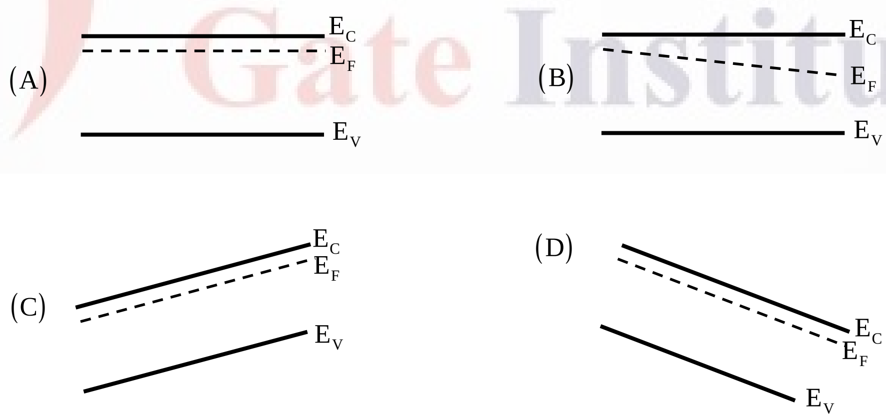
$$V_{BE_1} - V_{BE_2} = V_T \ln \frac{I_{S_2}}{I_{S_1}} = 26\times10^{-3} \ln\left[\frac{300\times300}{0.2\times0.2}\right] \quad \because I_S \alpha A$$

$$(V_{BE_1} - V_{BE_2}) = 381\text{mV}$$

36. An N-type semiconductor having uniform doping is biased as shown in the figure.

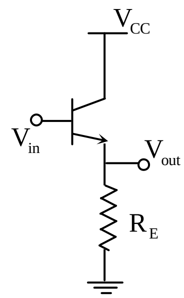


If E_C is the lowest energy level of the conduction band, E_V is the highest energy level of the valance band and E_F is the Fermi level, which one of the following represents the energy band diagram for the biased N-type semiconductor?



Answer: D

37. Consider the common-collector amplifier in the figure (bias circuitry ensures that the transistor operates in forward active region, but has been omitted for simplicity). Let I_C be the collector current, V_{BE} be the base-emitter voltage and V_T be the thermal voltage. Also, g_m and r_0 are the small-signal transconductance and output resistance of the transistor, respectively. Which one of the following conditions ensures a nearly constant small signal voltage gain for a wide range of values of R_E?



(A) $g_m R_E << 1$ (B) $I_C R_E >> V_T$ (C) $g_m r_0 >> 1$ (D) $V_{BE} >> V_r$

Answer: B

Exp: $$A_V = \frac{R_E}{r_e + R_E} = \frac{R_E}{\frac{V_T}{I_E} + R_E} = \frac{I_E R_E}{V_T + I_E R_E}$$

$$\therefore A_V \simeq \frac{I_C R_E}{V_T + I_C R_E} \quad (\because I_C \simeq I_E)$$

$$\therefore I_C R_E >> U_T \Rightarrow A_V \text{ in almost constant}$$

38. A BJT in a common-base configuration is used to amplify a signal received by a 50Ω antenna. Assume kT/q = 25 mV. The value of the collector bias current (in mA) required to match the input impedance of the amplifier to the impedance of the antenna is________.

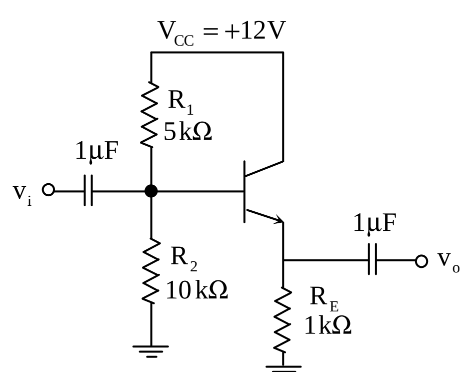
Answer: 0.5

Exp: Input impedance of CB amplifier, $z_i = r_e = \frac{V_I}{I_E}$

$$\Rightarrow 50 = \frac{25\,mV}{I_E} (\because \text{signal is received from } 50\Omega \text{ antenna and } V_T = 25\,mV)$$

$$\Rightarrow I_E = \frac{25\,mV}{50\Omega} = 0.5\,mA$$

39 . For the common collector amplifier shown in the figure, the BJT has high β, negligible $V_{CE(sat)}$, and V_{BE} = 0.7 V. The maximum undistorted peak-to-peak output voltage v_o (in Volts) is______.



Answer: 9.4

Exp: $\because \beta = \text{high}, I_B$ is neglected

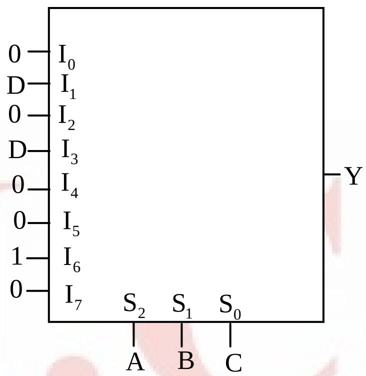
$$\therefore V_B = 12 \times \frac{10k}{10k + 5k} = 8V$$

$$V_E = V_B - 0.7 = 7.3V$$

$$\therefore V_{CE} = 12 - 7.3 = 4.7V$$

$$\therefore \text{Maximum undistorted } V_0 (p - p) = 2 \times 4.7V = 9.4V$$

40. An 8-to-1 multiplexer is used to implement a logical function Y as shown in the figure. The output Y is given by



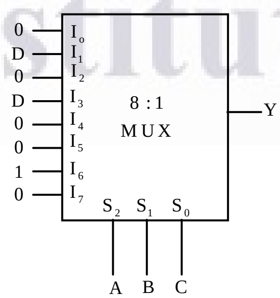
(A) $Y = A\bar{B}C + A\bar{C}D$ (B) $Y = \bar{A}BC + A\bar{B}D$

(C) $Y = ABC̄ + \bar{A}CD$ (D) $Y = \bar{A}\bar{B}D + A\bar{B}C$

Answer: C

Exp: $Y = \bar{A}\bar{B}CD + \bar{A}BCD + AB\bar{C}$

Remaining combinations of the select lines will produce output 0.

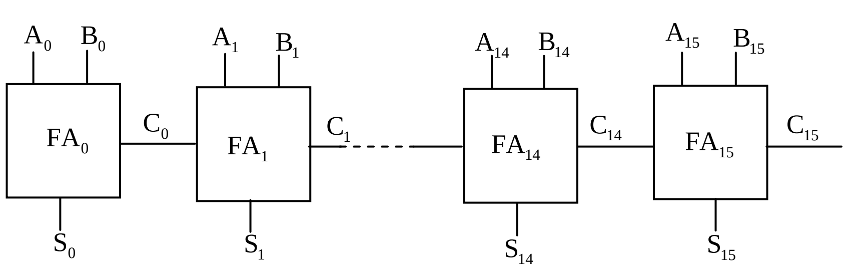


$$\text{So, } Y = \bar{A}CD(\bar{B} + B) + AB\bar{C}$$

$$= \bar{A}CD + AB\bar{C}$$

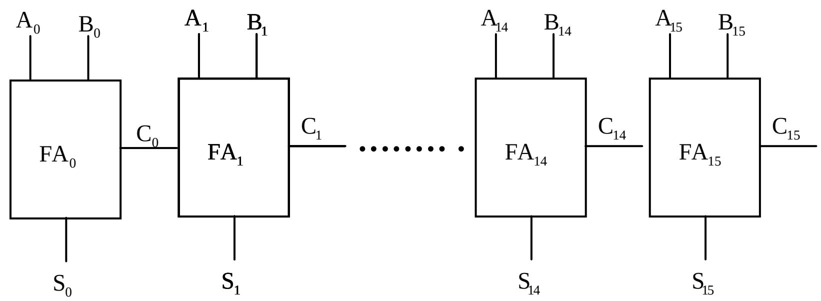
$$= AB\bar{C} + \bar{A}CD$$

41. A 16-bit ripple carry adder is realized using 16 identical full adders (FA) as shown in the figure. The carry-propagation delay of each FA is 12 ns and the sum-propagation delay of each FA is 15 ns. The worst case delay (in ns) of this 16-bit adder will be __________.



Answer: 195

Exp:



This is 16-bit ripple carry adder circuit, in their operation carry signal is propagating from 1st stage FA0 to last state FA15, so their propagation delay is added together but sum result is not propagating. We can say that next stage sum result depends upon previous carry.

So, last stage carry (C_{15}) will be produced after $16\times12\,\text{ns} = 192\,\text{ns}$

Second last stage carry (C_{14}) will be produced after 180 ns.

For last stage sum result (S_{15}) total delay = 180ns + 15ns = 195ns

So, worst case delay = 195 ns

42. An 8085 microprocessor executes "STA 1234H" with starting address location 1FFEH (STA copies the contents of the Accumulator to the 16-bit address location). While the instruction is fetched and executed, the sequence of values written at the address pins A_{15}-A_8 is

(A) 1FH, 1FH, 20H, 12H
(B) 1FH, FEH, 1FH, FFH, 12H
(C) 1FH, 1FH, 12H, 12H
(D) 1FH, 1FH, 12H, 20H, 12H

Answer: A

Exp: Let the opcode of STA is XXH and content of accumulator is YYH.

Instruction: STA 1234 H

Starting address given = 1FFEH

So, the sequence of data and addresses is given below:

Address (in hex) : Data (in hex)

$A_{15}-A_8$	A_7-A_0		Data
1F	FE H	$\rightarrow$	XXH
1F	FF H	$\rightarrow$	34H
20	00 H	$\rightarrow$	12 H
12	34 H	$\rightarrow$	YYH

43. A stable linear time invariant (LTI) system has a transfer function $H(s)=\dfrac{1}{s^2+s-6}$. To make this system causal it needs to be cascaded with another LTI system having a transfer function $H_1(s)$. A correct choice for $H_1(s)$ among the following options is

(A) s + 3
(B) s - 2
(C) s - 6
(D) s + 1

Answer: B

Exp: Given, $H(s)=\frac{1}{s^2+s-6}=\frac{1}{(s+3)(s-2)}$

It is given that system is stable thus its ROC includes $j\omega$ axis. This implies it cannot be causal, because for causal system ROC is right side of the rightmost pole.

$\Rightarrow$ Poles at $s=2$ must be removes so that it can be become causal and stable simultaneously.

$$\Rightarrow \frac{1}{(s+3)(s-2)}(s-2)=\frac{1}{s+3}$$

Thus $H_1(s)=s-2$

44. A causal LTI system has zero initial conditions and impulses response *h*(*t*). Its input *x*(*t*) and output *y*(*t*) are related through the linear constant-coefficient differential equation

$$\frac{d^2y(t)}{dt^2}+a\frac{dy(t)}{dt}+a^2y(t)=x(t)$$

Let another signal g(t) be defined as

$$g(t)=a^2\int_0^t h(\tau)d\tau+\frac{dh(t)}{dt}+ah(t)$$

If G(s) is the Laplace transform of g(t), then the number of poles of G(s) is _______.

Answer: 1

Exp: Given differential equation

$$s^2y(s)+\alpha\, sy(s)+\alpha^2y(s)=x(s)$$

$$\Rightarrow \frac{y(s)}{x(s)}=\frac{1}{s^2+\alpha s+\alpha^2}=H(s)$$

$$g(t)=\alpha^2\int_0^t h(z)dz+\frac{d}{dt}h(t)+\alpha h(t)$$

$$=\alpha^2\frac{H(s)}{s}+SH(s)+\alpha H(s)$$

$$=\alpha^2\frac{1}{s(s^2+\alpha s+\alpha^2)}+s\frac{1}{(s^2+2s+\alpha^2)}+\frac{\alpha}{s^2+\alpha s+\alpha^2}$$

$$=\frac{\alpha^2+\alpha s+s^2}{s(s^2+\alpha s+\alpha^2)}=\frac{1}{s}$$

No. of poles $=1$

45. The N-point DFT X of a sequence $x[n], 0 \le n \le N-1$ is given by

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}, \quad 0 \le k \le N-L$$

Denote this relation as X = DFT(x). For N = 4, which one of the following sequences satisfies DFT(DFT (x))=x ?

(A) $x = [1\ 2\ 3\ 4]$ (B) $x = [1\ 2\ 3\ 2]$

(C) $x = [1\ 3\ 2\ 2]$ (D) $x = [1\ 2\ 2\ 3]$

Answer: B

Exp: This can be solve by directly using option and satisfying the condition given in question

$X = DFT(x)$

$$D_{FT}(D_{FT}(x)) = DFT(X) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X[n] e^{-j\frac{2\pi}{N}nk}$$

DFT y $[1\ 2\ 3\ 4]$

$$X = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} 10 \\ 2+2j \\ 2 \\ -2-j2 \end{bmatrix}$$

DFT of (x) will not result in $[1\ 2\ 3\ 4]$

Try with DFT of y $[1\ 2\ 3\ 2]$

$$X = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \\ -1 \end{bmatrix}$$

$$\text{DFT of} \begin{bmatrix} 4 \\ - \\ 0 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 0 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

Same as x

Then 'B' is right option

46. The state transition matrix $\phi(t)$ of a system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(A) $\begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}$ (D) $\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

Answer: D

Exp: Given state model,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$\phi(t) \Rightarrow$ state transistion matrix

$$\phi(t) = L^{-1}\left[(SI - A)^{-1}\right]$$

$$[SI - A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} \Rightarrow \frac{1}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$$

$$\phi(t) = L^{-1}\begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

47. Consider a transfer function $G_p(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$ with p a positive real parameter. The maximum value of *p* until which G_P remains stable is ________.

Answer: 2

Exp: Given $G_p(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$

By R – H criteria

The characteristic equation is $s^2 + (3+p)s + (2-p) = 0$

i.e. $s^2 + (3+p)s + (2-p) = 0$

By forming R-H array,

$$\begin{array}{c|cc} s^2 & 1 & (2-p) \\ s^1 & (3+\phi) & 0 \\ s^0 & (2-p) & \end{array}$$

For stability, first column elements must be positive and non-zero

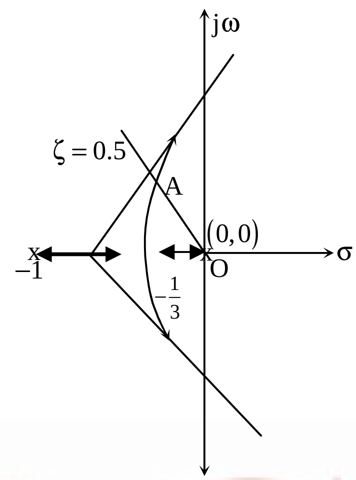
i.e. $(1)(3+p) > 0 \Rightarrow \boxed{p > -3}$

and $(2)(2-p) > 0 \Rightarrow \boxed{p < 2}$

i.e. $\boxed{-3 < p < 2}$

The maximum value of p unit which G_p remains stable is 2

48. The characteristic equation of a unity negative feedback system 1 + KG(s) = 0. The open loop transfer function G(s) has one pole at 0 and two poles at -1. The root locus of the system for varying K is shown in the figure.



The constant damping ratio line, for $\zeta = 0.5$, intersects the root locus at point A. The distance from the origin to point A is given as 0.5. The value of K at point A is ________ .

Answer: 0.375

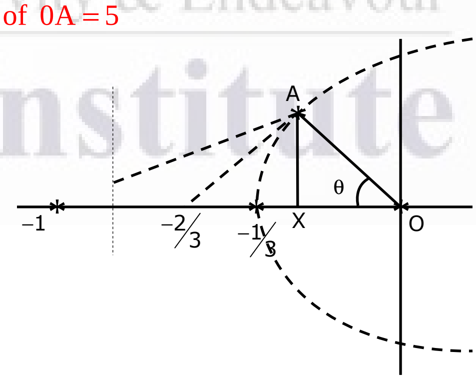
Exp: We know that the co-ordinate of point A of the given root locus i.e., magnitude condition

$|G(s)H(s)| = 1$

Here, the damping factor $\xi = 0.5$ and the length of $0A = 5$

$\xi = 0.5$

Then in the right angle triangle



$$\cos\theta = \frac{OX}{OA} \Rightarrow \cos 60 = \frac{OX}{0.5} \Rightarrow OX = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{AX}{OA} \Rightarrow \sin 60 = \frac{AX}{0.5} \Rightarrow AX = \frac{\sqrt{3}}{4}$$

So, the co-ordinate of point A is $-1/4 + j\sqrt{3}/4$

Substituting the above value of A in the transfer function and equating to 1

i.e. by magnitude condition,

$$\left| \frac{k}{s(s+1)^2} \right|_{s=-1/4+j\sqrt{3}/4} = 1$$

$$k = \sqrt{\frac{1}{16}+\frac{3}{16}} \cdot \left(\sqrt{\frac{9}{16}+\frac{3}{16}}\right)^2$$

$$\boxed{k = 0.375}$$

49. Consider a communication scheme where the binary valued signal X satisfies $P\{X = +1\}=0.75$ and $P\{X = -1\}= 0.25$. The received signal $Y = X + Z$, where Z is a Gaussian random variable with zero mean and variance σ^2. The received signal Y is fed to the threshold detector. The output of the threshold detector $\hat{X}$ is:

$$\hat{X}=\begin{cases}+1. & Y>\tau \\ -1. & Y\le\tau.\end{cases}$$

To achieve a minimum probability of error $P\{\hat{X}\neq X\}$, the threshold τ should be

(A) Strictly positive

(B) Zero

(C) Strictly negative

(D) Strictly positive, zero, or strictly negative depending on the nonzero value of σ^2

Answer: C

Exp: C

$H_1 : x = +1;\ H_0 : x = -1$

$P(H_1) = 0.75;\ P(H_0) = 0.25$

Received signal $\gamma = X + Z$

Where $Z \sim N(0,-2);\ \ f_Z(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-Z^2/2\sigma^2}$

$$\text{Received signal } \gamma = \begin{cases} 1+Z & \text{if } X=1 \\ -1+Z & \text{if } X=-1 \end{cases}$$

$$f_\gamma(y/H_1) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\gamma-1)^2}$$

$$f_\gamma(y/H_0) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\gamma+1)^2}$$

At optimum threshold y_{opt}: for minimum probability of error

$$\left.\frac{f_\gamma(y/H_1)}{f_\gamma(y/H_0)}\right|_{y=y_{opt}} = \frac{P(H_0)}{P(H_1)}$$

$$\left. e^{-\frac{1}{2\sigma^2}\left[(\gamma-1)^2-(\gamma+1)^2\right]}\right|_{y_{opt}} = \frac{P(H_0)}{P(H_1)}$$

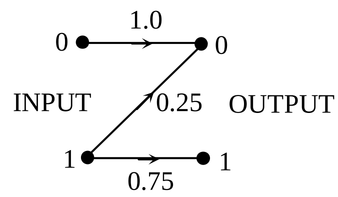
$$e^{+2y_{opt}/\sigma^2} = \frac{P(H_0)}{P(H_1)}$$

$$y_{opt} = \frac{\sigma^2}{2} l_n\left(\frac{P(H_0)}{P(H_1)}\right) = \frac{-1.1\sigma^2}{2} = -0.55\sigma^2$$

y_{opt} = Optimum threshold

$y_{opt} < 0 \ \therefore$ Threshold is negative.

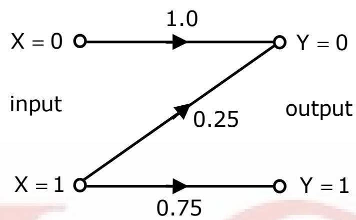
50. Consider the Z-channel given in the figure. The input is 0 or 1 with equal probability.



If the output is 0, the probability that the input is also 0 equals ______________

Answer: 0.8

Exp: Given channel



We have to determine, $P\{x=0/y=0\}$

$$P\{x=0/y=0\}=\frac{P\{y=0/x=0\}P\{x=0\}}{P\{y=0\}}=\frac{1.\frac{1}{2}}{1.\frac{1}{2}+0.25\times\frac{1}{2}}=\frac{4}{5}=0.8$$

51. An M-level PSK modulation scheme is used to transmit independent binary digits over a band-pass channel with bandwidth 100 kHz. The bit rate is 200 kbps and the system characteristic is a raised-cosine spectrum with 100% excess bandwidth. The minimum value of M is ________.

Answer: 16

Exp: Bandwidth requirement for m-level PSK $=\frac{1}{T}(1+\alpha)$

[Where T is symbol duration. α is roll of factor]

$$\Rightarrow \frac{1}{T}(1+\alpha)=100\times10^{3}$$

$\alpha=1$ [100% excess bandwidth]

$$\Rightarrow \frac{1}{T}(2)=100\times10^{3}$$

$$\Rightarrow T=\frac{2}{100\times10^{3}}=20\mu\sec$$

Bit duration

$$=\frac{1}{200\times10^{3}}=0.5\times10^{-5}=5\times10^{-6}\sec$$

$$\text{Bit duration}=\frac{\text{Symbol duration}}{\log_2 m}\Rightarrow \log_2 m=\frac{20\times10^{-6}\sec}{5\times10^{-6}}=4\Rightarrow M=16$$

52. Consider a discrete-time channel Y =- X + Z, where the additive noise z is signal-dependent. In particular, given the transmitted symbol $X \in \{-a, +a\}$. at any instant, the noise sample Z is chosen independently from a Gaussian distribution with mean βX and unit variance. Assume a threshold detector with zero threshold at the receiver.

When $\beta = 0$, the BER was found to be Q(a) = 1 × 10^{-8}

$$\left(Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\infty} e^{-u^2/2} du, \text{ and for } v > 1, \text{use } Q(v) \approx e^{-v^2/2}\right)$$

When $\beta = -0.3$, the BER is closest to

(A) 10^{-7} (B) 10^{-6} (C) 10^{-4} (D) 10^{-2}

Answer: C

Exp: X∈[-a,a] and P(x = -a) = P(x = a) = ½

γ=X+Z $\to$ Received signal

$$Z \sim N(\beta X, 1) \qquad Q(a) = 1\times10^{-8}$$

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(Z-\beta X)^2} \qquad Q(a) \approx e^{-\vartheta^2/2}$$

$$\gamma = \begin{cases} -a + z & \text{if } x = -a \\ a + z & \text{if } x = +a \end{cases}$$

$H_1 : x = +a$

$H_0 : x = -a$

and Threshold = 0

$$f_\gamma(y/H_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-a(1+\beta))^2}$$

$$f_\gamma(y/H_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y+a(1+\beta))^2}$$

BER :

$$P_e = P(H_1)P(e/H_1) + P(H_0)P(e/H_0)$$

$$= \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-a(1+\beta))^2} dy + \frac{1}{2}\int_{-0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y+a(1+\beta))^2} dy = Q(a(1+\beta))$$

$\beta = 0$

$$P_e = Q(a) = 1\times10^{-8} = e^{-a^2/2} \Rightarrow a = 6.07$$

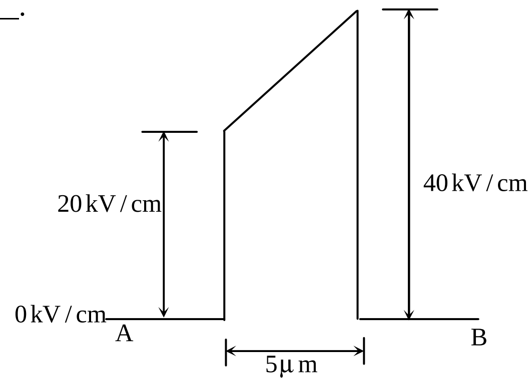
$\beta = -0.3$

$$P_e = Q(6.07(1-0.3)) = Q(4.249)$$

$$P_e = e^{-(4.249)^2/2} = 1.2\times10^{-4}$$

$$P_e \simeq 10^{-4}.$$

53. The electric field (assumed to be one-dimensional) between two points A and B is shown. Let ψ_A and ψ_B be the electrostatic potentials at A and B, respectively. The value of $\psi_B - \psi_A$ in Volts is ________.



Answer: -15

Exp: A B

$(0\,kV/cm,\ 20\,kV/cm)$ $(5\times10^{-4}\,kV/cm,\ 40\,kV/cm)$

$$E-20=\frac{40-20}{5\times10^{-4}}(x-0)\Rightarrow E=4\times10^{4}x+20$$

$$V_{AB}=-\int_A^B E.dl=-\int_0^{5\times10^{-4}/cm}\left(4\times10^4x+20\right)dx$$

$$=-\left(4\times10^4\frac{x^2}{2}+20x\right)\Bigg|_0^{5\times10^{-4}}=-\left(2\times10^4\times25\times10^{-8}+20\times5\times10^{-4}\right)$$

$$=-\left(50\times10^{-4}+100\times10^{-4}\right)=-150\times10^{-4}\,kV$$

$$\Rightarrow V_{AB}=-15V$$

54. Given $\vec{F}=z\hat{a}_x+x\hat{a}_y+y\hat{a}_z$. If S represents the portion of the sphere $x^2+y^2+z^2=1$ for $z\ge0$, then $\int_S \nabla\times\vec{F}\,.\,\vec{ds}$ is ___________.

Answer: 3.14

Exp: $\int_S \nabla\times\vec{F}.d\vec{s}=\oint_C \vec{F}.d\vec{r}$ (using stoke's theorem and C is closed curve i.e.,

$$x^2+y^2=1,\ z=0$$

$$\Rightarrow x=\cos\theta, y=\sin\theta \text{ and } \theta:0 \text{ to } 2\pi$$

$$=\oint_C zdx+xdy+ydz$$

$$=\oint_C xdy=\int_0^{2\pi}\cos\theta(\cos\theta\,d\theta)$$

$$=\frac{1}{2}\left(\theta+\frac{\sin2\theta}{2}\right)_0^{2\pi}=\pi\simeq3.14$$

55. If $E = -\left(2y^3 - 3yz^2\right)\hat{x} - \left(6xy^2 - 3xz^2\right)\hat{y} + \left(6xyz\right)\hat{z}$ is the electric field in a source free region, a valid expression for the electrostatic potential is

(A) $xy^3 - yz^2$ (B) $2xy^3 - xyz^2$ (C) $y^3 + xyz^2$ (D) $2xy^3 - 3xyz^2$

Answer: D

Exp: Given $E = -\left(2y^3 - 3yz^2\right)a_x - \left(6xy^2 - 3xz^2\right)a_y + 6xyz.a_z$

By verification option (D) satisfy

$E = -\nabla V$